Optimal Design of Rolling-Contact Bearings Via Evolutionary Algorithms

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Abstract: *RKB Bearing Industries Group has been using Evolutionary Algorithms for long, well aware that optimized products can make a difference in performance compared to the other producers. This paper is a brief review of the main aspects of the optimal design and a report on the achievements of the RKB Advanced Calculations and Optimization Department in the field. The differences between optimal design and conventional design are pointed out by way of a very simple example of mechanical design. Multi-objective optimal design via Evolutionary Algorithms of a specific cylindrical roller bearing is also presented.*

Key words: Optimization, Bearing, *Evolutionary algorithms, Optimal design*

1. Optimal design in modern engineering

Optimization is an important concept in engineering. Finding any solution to a problem is not nearly as good as finding the one *optimal solution to the problem*. In the last decades the complexity of conceived products have met an extraordinary growth and it is estimated that, in the near future, designing a product will have to take into consideration a multitude of factors including actual design, manufacturing and logistics (supplying and distribution). The ever growing complexity of design problems obviously requires appropriate instruments. The present tendency in technical design of products is *optimal design, which means conceiving and solving a mathematical programming problem based on the mathematical model of a real engineering problem.*

2. Structure of mathematical programming problems

In mathematics, *optimization*, or *mathematical programming*, refers to choosing the best element from some set of available alternatives. Often, this number of possible alternatives is infinite or, at least, very high in computational time terms. It is worth noting here that the term *mathematical programming* is not directly related to computer programming.

A mathematical programming problem has to fit to a certain format. Let the decision (design) variable vector be:

$$\overline{x} = \left(x_1, x_2, \dots, x_p\right)$$

and suppose that its components are laying in the ranges:

$$x_1 \in \left[x_1^l, x_1^u\right] \;,\; x_2 \in \left[x_2^l, x_2^u\right] \;, \dots ,\; x_p \in \left[x_p^l, x_p^u\right]$$

Consider also the objective function (mono-objective optimization) or objective functions (multi-objective optimization) and a set of constraints (all of these are functions of the decision vector $\overline{x}$). Note that mono-objective optimization and multi-objective optimization respectively are two totally different approaches as will be seen in the following pages. To solve this mathematical programming problem means that one has to find $\overline{x}$ so that:

$$\textbf{objective function(s):} \begin{cases} f_1(\overline{x}) & \rightarrow \quad \textbf{min} \text{ or } \textbf{max} \\ f_2(\overline{x}) & \rightarrow \quad \textbf{min} \text{ or } \textbf{max} \\ \dots\dots\dots\dots\dots\dots & \\ f_m(\overline{x}) & \rightarrow \quad \textbf{min} \text{ or } \textbf{max} \end{cases}$$

$$\textbf{constraints:} \begin{cases} g_1(\bar{x}) \lhd 0 \\ g_2(\bar{x}) \lhd 0 \\ \cdots\cdots\cdots \\ g_q(\bar{x}) \lhd 0 \end{cases}$$

where the sign " $\lhd$ " should be read "less than" or "less than or equal to" or even "equal to" as appropriate.

3. Optimal design versus conventional design

To fully understand the fundamental differences between conventional and optimal design will address a simple statement of the design of machine parts. The main problem in the field of mechanical design work is the dimensioning (sizing) of machine parts according to some known requirements. Sometimes you can also read of "pre-dimensioning", that is determination of the main dimensions of the required mechanical parts by means of simplified relationships.

Essentially, sizing means to solve an equation that describes the equality between a stress and an allowable stress. This equation has a certain number of unknowns (usually geometric dimensions) and, of course, has infinity of solutions. To solve such an equation the designer is forced to choose only one of these unknowns (main unknown) and consider as known all the others (giving them concrete values using its own experience and indications in the literature) or to express them as a function of the main unknown. Often the unknown what is meant to be removed is expressed as a product of the main unknown and a coefficient for which there are indications (within limits, sometimes very large) in the literature. In this way, a single-unknown equation is obtained and solved without any difficulty.

Unfortunately, such an approach represents only the solving of the initial problem within a hyper-plane of the solutions space of the equation with several unknowns. There is no guarantee that this solution is the best (optimum) of the potential solutions to the problem. In addition, the solution found by solving an equation with a single unknown might not agree in other respects that the designer has not taken or could not take into account when writing the sizing equation. The constraints that one has to take into consideration refer to the economic, technological, assembling, material aspects of the design problem.

It should be also discussed another aspect of the problem. In recent years there has been an explosive development of CAD software tools that enable a thorough analysis of the state of stresses and strains occurring in different designed parts. CAD software provides designer with particularly strong tools, but to start using any of these applications an "object" with a certain geometric shape and size is required. But this is precisely the question: How and where did these dimensions and this shape come from? The answer offered by the conventional design is often a "covering" pre-sizing and, therefore, the analysis of the state of stress and strain not infrequently finds that the designed object should be further modified to make better use of the available material. In this situation it is necessary to amend, by means of a CAD application, the size and sometimes the shape of the designed part in order to near the actual values of the stresses and deformations to the allowable ones. This process is time and skilled human resources consuming and, obviously, does not guarantee the best design solution in the given context. The answer given by the optimal design is an object that requires infinitely less subsequent changes in shape and size of the designed piece (or removes them completely), thereby allowing the designer, on the one hand, to make some fine adjustments, and, on the other, to identify and approach other design issues (all related to the discussed object), which could otherwise remain hidden or neglected.

All the above clearly shows the optimal design advantages compared with conventional design. In the optimal design, the correct formulation of the mathematical programming problem will perform the coagulation of all project aspects in a uniform and global picture. All that, in classical design, was a succession of phases becomes a single process when the chosen approach is the optimal design. One might say that now all these phases are executed concurrently.

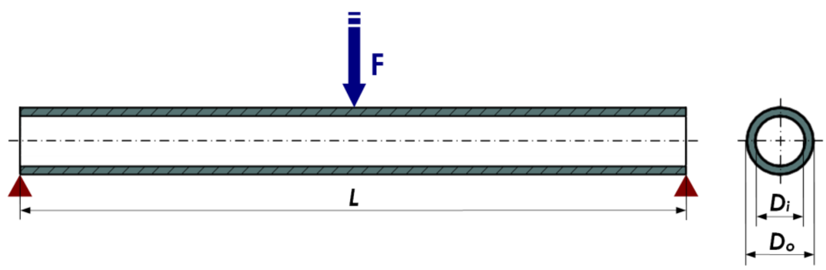


Fig. 1 - *Tubular beam design*

Let's illustrate the above considerations by approaching a very simple design problem: the tubular beam of length *L* in figure 1 is centrally loaded by force *F*. Knowing the material of the beam, the problem is to find the values of outer diameter D_o and inner diameter D_i of the beam. The only equation available for dimensioning is the well-known bending stress equation:

$$\sigma_b = \frac{16 \cdot F \cdot L \cdot D_o}{\pi \cdot \left(D_o^4 - D_i^4\right)}$$

Since there are two unknowns (D_o and D_i) and only one equation, in conventional design, one of the unknowns (D_i in this case) is expressed as a function of the other unknown, using a coefficient of proportionality *k*:

$$D_i = k \cdot D_o$$

The equality between the bending stress and the allowable bending stress yields

$$D_o = \sqrt[3]{\frac{16 \cdot F \cdot L}{\pi \cdot \sigma_{b\_allow} \cdot \left(1 - k^4\right)}}$$

and consecutively the value of the inner diameter is obtained according to the imposed proportionality. For example, if F = 10,000 N, L = 1,000 mm and the material of the beam is steel with density $\rho = 7.87 \cdot 10^{-6}$ kg/mm^3 and allowable bending stress σ_{b_allow} = 100 MPa and if one chooses a value k = 0.75 for the coefficient of proportionality (from a literature recommended range of 0.7 to 0.85), one obtains D_o = 91 mm and D_i = 68 mm (rounded to integer values). In this case the mass of the beam is about 22.6 kg. Obviously the key feature of this design calculation is the selection of the coefficient *k* value. In fact from 0.7 to 0.85 the designer can select any value (unless additional aspects about the design are known) and obtains different results for D_o and D_i.

In case of the optimal design approach, the designer can focus on any aspect that seems important to him. If, in the above case of tubular beam, his aim is to obtain a beam as lighter as possible in the given context, the mass of the beam will be the objective function and all the other design aspects he intends to take into consideration will form the constraint set. In this way he establishes the mathematical programming problem:

- *Objective function:* mass of the beam

$$M(D_o, D_i) = \rho \cdot \frac{\pi \cdot \left(D_o^2 - D_i^2\right)}{4} \cdot L \quad \rightarrow \quad \textbf{min}$$

- *Constraints:*

1. The bending stress should be less than or equal to the allowable bending stress:

$$\sigma_b \le \sigma_{b\_allow} \quad \Rightarrow \quad g_1(D_o, D_i) = \sigma_b - \sigma_{b\_allow} \le 0$$

2. The thickness of the tubular beam wall should be at least a certain value δ (we consider δ = 6 mm in our example):

$$D_e - D_i \ge 2 \cdot \delta \quad \Rightarrow \quad g_2(D_o, D_i) = 2 \cdot \delta - D_e + D_i \le 0$$

Solving this mathematical programming problem yields the following values: D_o = 112 mm, D_i = 100 mm and beam mass = 15.7 kg, which represents a decreasing by about 32% in comparison with the mass of the same beam obtained by conventional design.

Certainly, the above example is very simple but our goal was only to prove how a conventional design problem can be transformed into an optimal design one.

4. Conceiving the mathematical programming problem

After a rigorous study of the requirements for the specific product (for example a rolling-contact bearing or any other machine element), a set of independent parameters that fully define the product can be identified (type, geometrical dimensions, materials etc.). These parameters are in fact the *unknown variables* of the design problem and also of the optimization program. The optimization program will consist of one or more objective functions that have to be either maximized or minimized and a set of constraints.

Establishing of the objective function(s) is the most important step in setting-up the optimization process and directly results from the problem or has to be deducted from the general requirements. When these requirements are multiple and even conflicting, a weighted objective function or a multi-objective optimization algorithm can be used. A typical case would be that when one wants high performance, while aiming to reduce, as much as possible, the costs. Objective functions can be the mass of single parts or subassemblies, radial or axial basic dynamic load rating of bearings, minimum thickness of the elastohydrodynamic lubricant film between the rolling element and bearing raceway, deformations of components of construction, potential energy stored, efficiency of transmission etc.

A very important step in building an optimization problem is to determine the ranges where the design variables can take values, since the Cartesian product of these ranges becomes the search space (where one intends to find the optimal solution). This operation requires both engineering intuition and awareness of the limits of the chosen method. If the search space is too large, the probability of convergence in real time to a minimum (or maximum) of the objective function decreases, and if it is too narrow there is danger of losing solutions, some of which "unexpected", but sometimes very useful.

Another key step in conceiving an optimal design problem is the setup of the constraint set. The more accurately the constraints reflect the actual design requirements, the closer to the design ideal solution the obtained solution is, but unfortunately the more difficult to solve the programing problem is. As constraints one can use verification relations of machine elements, technical or economic restrictions and many other restrictive but real conditions.

It is worth mentioning that the objective function and the constraints should be considered in a wider meaning, namely as *procedures*.

At this point two issues have to be solved:

- how many objective functions should we use?
- which are the instruments (algorithms) to be used for solving the optimization program (optimal design problem)?

The first issue is closely connected to the specific optimal design problem and is up to the designer to choose the appropriate approach. For the second question we think that the answer lies in the so-called *Evolutionary Algorithms*.

5. Mono-objective and multi-objective optimization

Many researches in the field of engineering optimization (optimal design) began with mono-objective optimization, reaching notable results. This was an important step toward the approach of multi-objective optimization, given the fact that most real-world engineering problems have several objectives that are often in conflict. For example, if one refers to the optimization of a certain structure, one will normally want to minimize its weight (and consequently minimize its volume) and, at the same time, to minimize its deformations, so as to provide the maximum possible safety. These objectives are, however, conflicting, since minimal deformations will require a higher volume of material and, consequently, a higher weight.

Another example (Deb, 2005) is that of the design of a product for minimum size and for maximum output (e.g. delivered power) simultaneously. Ideally, such a bi-objective optimization task results in a set of optimal solutions (known as Pareto-optimal solutions), each portraying a trade-off between the two objectives. Amongst these optimal solutions there is a solution (say solution A), which is the best for size consideration and hopefully a different solution (say solution B), which is the best for output consideration. Obviously, solution A has the smallest dimension possible, but it will often not be able to deliver much an output. On the other hand solution B (using the same technology) has a size and weight substantially large, but it is able to deliver the maximum output.

There may also be a host of other solutions which are not as good as A in terms of size or not as good as B in terms of delivered output, but these intermediate solutions are good compromises to solutions A and B. However, one is not simply interested in finding a set of such optimal trade-off solutions, but wants to find and analyze them to discover some interesting commonality principles among all or many of these optimal solutions (useful in a possible future standardization).

Since Pareto-based algorithms are probably the most suitable approaches for engineering optimal design, hereinafter we will preferably discuss only these ones. Pareto optimality was introduced in the late nineteenth century by the Italian economist Vilfredo Pareto, and is defined as follows: *A solution is said to be Pareto optimal if there exists no other solution that is better in all attributes (objectives).* This implies that in order to achieve a better value in one objective at least one of the other objectives is going to deteriorate if the solution is Pareto optimal. Thus, the outcome of a Pareto optimization is not one optimal point, but a set of Pareto optimal solutions (Pareto front) that visualize the trade-off between the objectives. A bi-objective Pareto front is roughly sketched in figure 2.

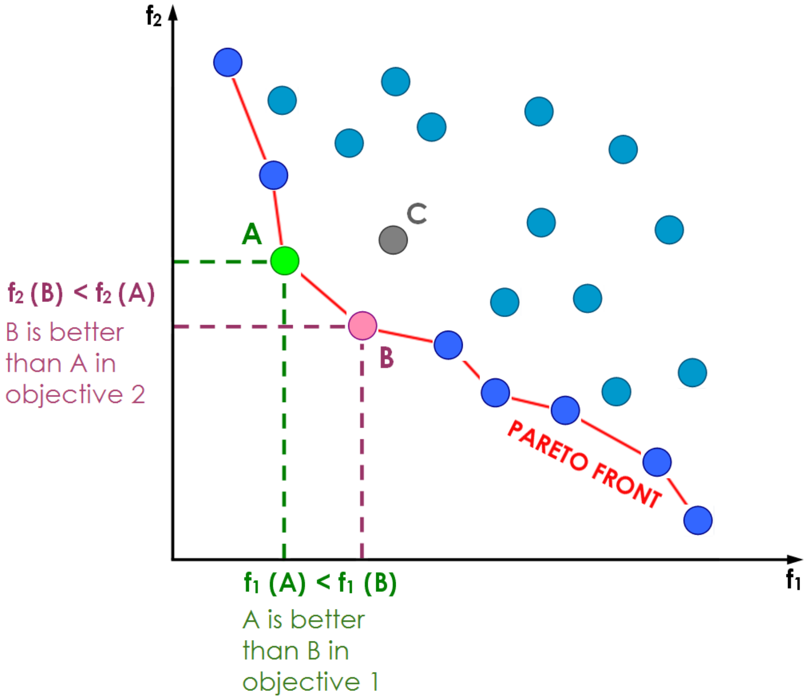


Fig. 2 - *Bi-objective Pareto front*

6. What are Evolutionary Algorithms?

If one goes for the optimal design option, the next step is to select the instrument for solving the mathematical programming problem. Most traditional techniques for the optimization of systems of several parameters involve the calculation of the first and second order partial derivatives of the objective function with respect to all variables. However, the situations where the analytical approach is difficult (large number of variables, very complex objective function, extremely large number of constraints) or impossible (discontinuous objective function, discrete variables etc.) are very common.

Another disadvantage of conventional optimization algorithms is represented by their rigidity. Change of the objective function, widening of the search space, adding, removal or modification of constraints in most cases suppose a complete rewriting of the method, which is unacceptable in a world where huge amounts or positions on the market may be lost due to a slow adaptation to changing conditions of production. Also, this lack of flexibility obstructs the user to effectively implement the algorithm to similar but different problems (for example in terms of objective function).

As often happens, nature seems to come to the aid of mathematicians and engineers, inspiring a new generation of optimization techniques: *evolutionary algorithms*. They take on natural selection principles and apply them to a population of possible solutions (called individuals or sometimes simply chromosomes) of an optimization problem. This is aimed at creating off-springs with "features" better than their parents, i.e. solutions that, if replaced in the objective function, will give better objective values.

Evolutionary Algorithms (EAs) are a subset of *Evolutionary Computation*, generic population-based metaheuristic optimization algorithms. EAs encompass many optimization techniques as Genetic Algorithms (GAs), Evolutionary Programming (EP), Evolution Strategy (ES) etc. In the last years, the boundaries between GAs, EP and ES have been broken down and nowadays EAs combine the advantages of all these approaches. Recently this optimization algorithm class was completed by the so-called Memetic Algorithms (MAs).

An EA uses some mechanisms inspired by the biological evolution: *natural selection*, *reproduction*, *mutation*, *recombination*, and *survival of the fittest*. Each optimization parameter x_i is encoded into a gene using binary or real codes. The genes of all parameters x_1, x_2, ... , x_p form a chromosome (individual) which describes a unique designing solution. A set of chromosomes, forming a set of distinct solutions, forms a population amongst which the best individuals are selected (based on the fitness function) for reproduction. Recombination is made using a specialized operator and the combination of the genes of the parents results in the offspring. The offspring (mutated or not) are inserted into the population and the described procedure starts again creating in this way an artificial Darwinian environment. The evolution of the population, therefore, takes place after the repeated application of the above operators. In figure 3 a very simple scheme of an EA is presented.

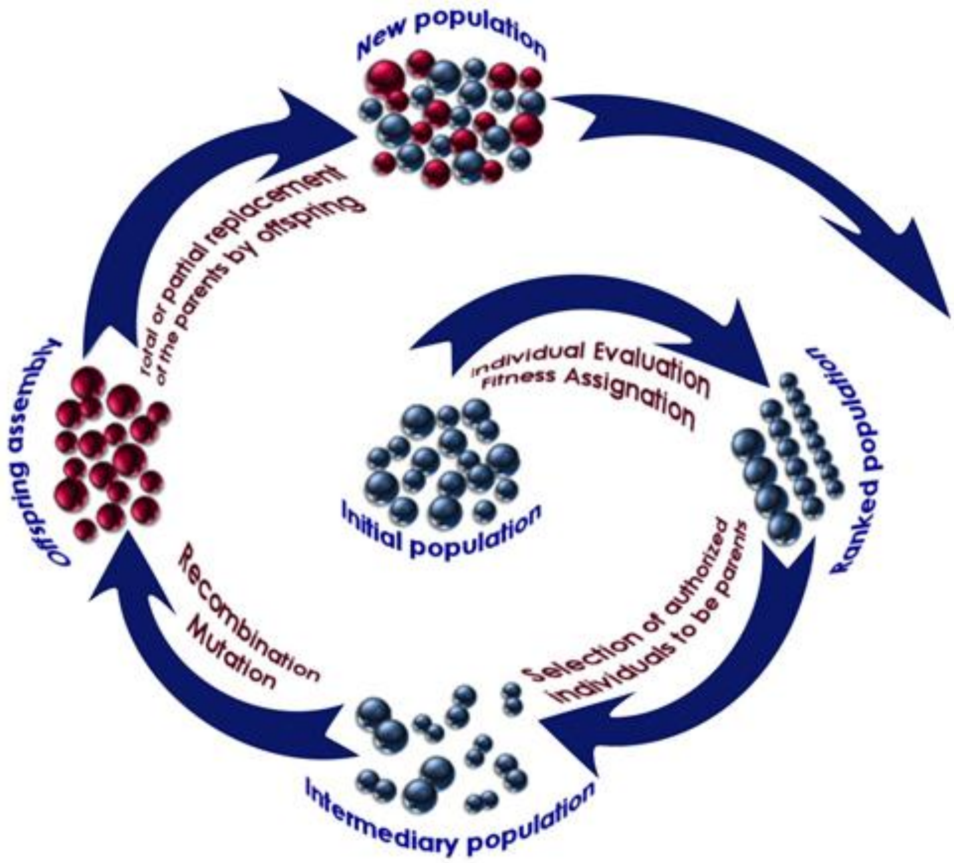


Fig. 3 - *Principle of a very simple Evolutionary Algorithm*

Since the simplest EAs are good at exploring the solution space (because they search from a set of designs and not from a single design solution), their important drawback is that they are not well suited to perform finely tuned search.

Starting from an idea of P. Moscato (1989) it has been argued (Hart, 1994; Wolpert, 1997; Culberson, 1998; Goldberg 1999) that it is essential to incorporate some form of domain knowledge into EAs to arrive at highly effective search. "Evolutionary Algorithms which include a stage of individual optimization or learning (usually in the form of Neighborhood Search - NS) as a part of their strategy are Memetic Algorithms" (Krasnogor, 2002).

MAs have shown to be orders of magnitude faster than traditional GAs for some problem domains, even in well-known large combinatorial optimization problems where other metaheuristics have failed. MAs are inspired by Dawkin's (1976) "meme" (abbreviation of the Greek word "mieme"), a concept representing a unit of cultural evolution, replicated by imitation, counterpart of "gene" in Darwinian evolution. It is worth noting that, so far, MAs have not been used yet in this sense, but this is the general trend in the field. As Krasnogor (2002) noted "there is much to be learned" in order to obtain real MAs.

In practice, MA is a GA modified as follows: in each generation of GA, the NS operator is applied to all solutions in the offspring population, before applying the selection operator.

7. Why Evolutionary Algorithms?

As Peter J. Bentley highlights in his excellent book - *Evolutionary Design by Computers*, 1999 – there are four main reasons why the choice of *Evolutionary Algorithms* is appropriate for design problems:

- evolution is a good, general-purpose problem solver;
- uniquely, *Evolutionary Algorithms* have been used successfully in every type of engineering design;
- evolution and the human design process share many similar characteristics. It is interesting to mention here that since 1991, David Goldberg compares the recombination of genetic material from parent solutions when forming a new child solution with a human designer combining ideas from two solutions to form a new solution;
- the most successful and remarkable designs known to mankind were created by natural evolution.

In order to easily use an optimization technique for complex (hard) optimal design problems, the algorithm should be a robust one. David Goldberg (1989) has defined robustness as "the balance between efficiency and efficacy necessary for survival in many different environments".

Then one can define two purposes in constructing an optimization technique as its *efficacy* and *efficiency*. *Efficacy* means whether the optimization technique can reach the optimum or not. The common purpose in constructing optimization techniques is this efficacy, that is their convergence to the optimum of the problem. The other purpose, *efficiency*, means whether the technique can find a better solution under the constraints the problem has. The technique may not find the optimal solution of the problem due to the constraints, but it is important that better solutions are searched by the algorithm within the constraints. Therefore Goldberg concluded that "the most important goal of optimization is improvement. (...) Attainment of the optimum is much less important for complex systems" (1989). Obviously, *Evolutionary Algorithms* totally fulfill all these requirements.

8. Multi-objective Evolutionary Algorithms (MOEAs)

The multi-objective optimization algorithms represent the state-of-the-art of all optimization algorithms. The first implementation of a MOEA dates back to the mid-1980s (Schaffer's Vector Evaluation Genetic Algorithm – VEGA). Since then, a considerable amount of research has been done in this area, now known as Evolutionary Multi-objective Optimization (EMO). As we already mentioned, since Pareto-based algorithms are probably the most suitable approaches for engineering optimal design, hereinafter we will preferably present these MOEAs. One can roughly divide MOEAs into two generations:

- *first generation* MOEAs typically adopt niching or fitness sharing in order to block premature convergence;
- the *second generation* of MOEAs was born with the introduction of the notion of elitism. In the context of multi-objective optimization, elitism usually (although not necessarily) refers to the use of an external population (also called *secondary population*) to retain the nondominated individuals. Elitism can also be introduced through the use of a (μ+λ)-selection in which parents compete with their offspring, and those that are nondominated (and possibly comply with some additional criterion such as providing a better distribution of solutions) are selected for the following generation.
  Second generation MOEAs can be characterized by an emphasis on efficiency and by the use of elitism (in the two main forms previously described). Today the transition from two to three objective functions is taking place in the literature, and the high-dimensional problems are the current focus of study among EMO research (Fleming, 2005).

9. Optimal design of rolling-contact bearings via Evolutionary Algorithms

Rolling bearings represent a perfect application field for Evolutionary Algorithms. Almost any area of today's industry uses rolling-contact bearings. An object that seems so trivial actually turns out to incorporate a huge amount of knowledge from mechanics, mathematics, physics, chemistry and the list seems endless. Among other things, the rolling element bearing industry was among the first to use fatigue life as a design criterion.

However, notwithstanding the leading-edge scientific knowledge at the base of the product, the bearing industry remained somewhat outside the concerns of professionals working in the field of optimization. In the last two decades the number of scientific papers dealing with optimal design of bearings (whatever the used optimization algorithms are) is substantially lower than that of optimal design works in other fields of engineering, although the results have been more than promising.

One of the first works in the field of optimal design of rolling-contact bearings is that of Boesiger and Warner (1991), which addresses the optimization of retainer spin bearing for momentum wheels (control-moment gyroscope, reaction wheel assembly). Wan Changsen, in his book *Analysis of Rolling Element Bearings* (1992), described an optimal design method by using a gradient-based numerical optimization technique for rolling-contact bearings. Probably for the first time he proposed five objective functions for optimal design of rolling-contact bearings:

- the maximum fatigue life;
- the maximum wear life;
- the maximum static load rating;
- the minimum frictional moment;
- the minimum spin to roll ratio.

These objective functions are nonlinear in nature and, moreover, they are associated with the geometric and kinematic constraints. Since the maximum fatigue life is the most important of the above objective functions (in fact fatigue failure is the main type of failure mode in rolling-contact bearings used in practical applications), Changsen proposed another single-objective function, namely basic dynamic load rating of bearings related to the bearing life. His formulation encompassed five design variables and inequality constraints for the optimum design of deep groove ball bearings. However, only the basic concepts and solution techniques of the optimization problem were introduced, without any illustrations and solutions for the formulation he obtained. It is worth noting here that the concept of multi-objective optimization of rolling-contact bearings was also proposed.

After more than ten years, Chakraborthy et al. (2003) employed a binary-coded genetic algorithm (BGA) to solve the Changsen's formulation based on the requirement of the longest fatigue life (in fact the objective function was a simplified form of the radial basic dynamic load rating). They used five design parameters (variables or genes in terms of EAs): ball diameter, number of balls, pitch diameter, curvature radius coefficient of the outer raceway groove, and curvature radius coefficient of the inner raceway groove. However, the assembling angles and boundaries of the inequality constraints used were unrealistic and led to unrealistic results.

Rao and Tiwari (2007) tried to correct this drawback and developed a rolling-contact bearing design methodology with improved and realistic constraints for the same single objective optimization with the help of GAs. In their approach the constraints contain unknown constants, which have been given ranges based on parametric studies through initial optimization runs. In the final run of the optimization, these constraint constants have been included as design parameters as well. The optimized design parameters have been found to yield better fatigue life as compared to those listed in standard catalogues.

Recently, Lin (2010), using a real-valued GA with differential evolution (DE) together with a proper and original handling of those 20 constraints, dramatically improved the previous results obtained by the above mentioned predecessors in the optimum design of deep groove ball bearings, based on maximum fatigue life as an objective function. The results clearly show that the so-called GA–DE algorithm can successfully find the best dynamic load ratings, about 1.3–11.1% higher than those obtained using the traditional BGA.

As for radial roller bearings, Kumar et al. (2008) developed a so-called optimum design methodology of cylindrical roller bearings using GAs. Also in this case the basic dynamic load rating was chosen as the objective function. Design variables included four geometrical parameters: the bearing pitch diameter, the diameter of the roller, the effective length of the roller, and the number of rollers. In addition, another five design constraint constants were included, which indirectly affect the bearing basic dynamic load capacity. These five design constraint constants were given bounds based on the parametric studies through the initial optimization runs. The optimization results proved a good agreement between the optimized and standard bearings in respect to the basic dynamic load rating.

As is known the profile (crowning) of the roller plays an important role in the increase of life of cylindrical roller bearings. A flat profile of the rolling element results in edge stress concentrations at roller ends. A circular crowning of the roller eliminates the edge stress concentrations at low and moderate loads, but develops edge stress concentrations at heavy loads. The logarithmic profile of the roller results in no edge stress concentration at low, medium, and heavy loads, and the distribution of contact stresses is nearly uniform along the length of the roller. In 2009 Kumar et al. improved this design methodology for the optimum design of cylindrical roller bearings by including the effect of the roller profile. Besides the already mentioned variable, they added two logarithmic profile generating parameters. Optimization results showed that the multiplier of the logarithmic profile deviation parameter has more effect on the fatigue life as compared with other geometric parameters.

In the context of roller profile influence on the bearing fatigue life, the important results obtained by Krzemiński-Freda and Warda (1996) and Fujiwara and Kawase (2006) have to be mentioned, even if the optimization was not performed via EAs.

Regarding the optimal design of rolling-contact bearings by means of MOEAs, one might say that it is only at the very beginning. One of the first approaches is that of Gupta et al. (2007), in which three primary objectives for a deep-grove ball bearing (namely the basic dynamic load rating, the basic static load rating and the elastohydrodynamic minimum film thickness) have been optimized separately, pair-wise and simultaneously using the well-known NSGA II multi-objective optimization algorithm (Deb, 2002). The same ten design variables and almost the same constraints as in Rao's 2007 work were used. Bi-objective and three-objective Pareto fronts revealed some important conclusions:

- the basic dynamic load rating and the basic static load rating are optimized simultaneously;
- trade-off fronts should be used for studying effects of various parameters behind the calculation of dynamic and static load ratings;
- one can perform a parametric study to find out the variation in the trade-off with the changing operating conditions;
- dynamic and static load ratings have been found to be very sensitive to variations in the inner raceway curvature coefficient.

In 2009 Savsani et al., using a modified particle swarm optimization technique, reported for the same problem much better results in comparison to those of Gupta, but the above conclusions remain valid.

In 2010 Wei and Chengzu accomplished the optimal design of a high speed angular contact ball bearing by using the same Deb's NSGA II multi-objective optimization algorithm. This time the objective functions were rating life and spin frictional power loss. The results were remarkable: the optimized maximum rating life of the 7007AC bearing operating at a DN (bore/mm × shaft speed/rpm) of 2.2 million was 7.5 times higher than that of conventional design, and when the spin frictional power loss was the same as in the case of conventional design, the rating life was 36% longer than that of the referential (current) design.

10. The use of MOEAs at RKB Bearing Industries

By way of example, we briefly present the multi-objective optimal design of the RKB NP 1092 cylindrical roller bearing (figure 4) accomplished by the Advanced Calculations and Optimization Department of the RKB Bearing Industries Group. The main geometric dimensions of this bearing are:

- nominal bore diameter d = 460 mm;
- nominal outer diameter D = 680 mm;
- nominal width B = 100 mm.

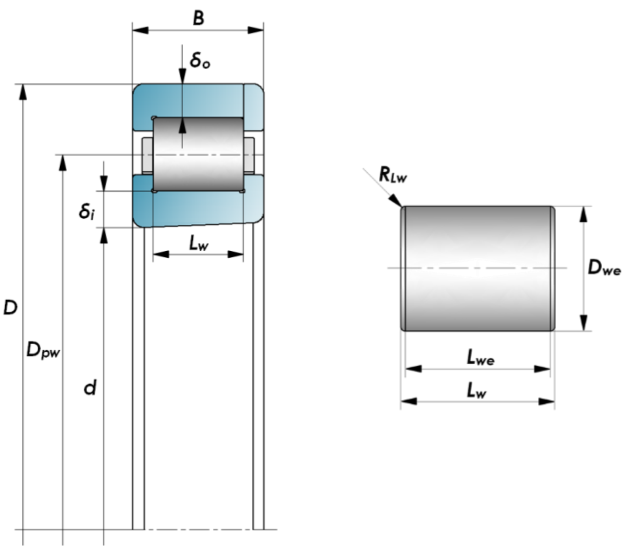


Fig. 4 - *NP1092 cylindrical roller bearing*

After a close study we concluded that the internal geometry of the bearing can be described by means of only three genes (table 1). Note that, as the optimization was performed using our optimization software based on Evolutionary Algorithms, the term variable is replaced by the specific term of *gene*. Unlike other previous works we set up a very simple algorithm to establish the appropriate number of rollers for each parameter set describing the internal geometry of the bearing. However, even if the roller length can be easily determined, we preferred to introduce L_w as a variable and write the program in the general case.

Table 1 - *Genes of the multi-objective optimal design of NP 1092 bearing*

No.	Gene	Denotation	Range	Type	Precision
1	Complex factor	γ	0.00001 … 0.3	real	10^{-6}
2	Roller diameter	D_{we}	1 … 100 mm	integer	10^{0}
3	Roller length	L_w	5 … 200 mm	integer	10^{0}

The objective functions chosen for the optimization were:

- basic dynamic radial load rating of the bearing;
- minimum lubricant film thickness in the area of rolling contacts (roller-inner ring raceway and roller-outer ring raceway).

Obviously both functions should be simultaneously maximized even if and the more they are in conflict. Since the calculation of the minimum lubricant film thickness in the area of rolling contacts involves some radial load, unlike other works in the field, we assumed that the radial force loading the bearing is fraction (denoted by p_C) of the basic dynamic radial load rating of the bearing. We considered a value of 0.15 for this fraction, considering it as the frontier between a loaded bearing and a very loaded bearing. According to these considerations, the objective functions become as follows:

$$C_r(\gamma, D_{we}, L_{we}) = f_{Cr}(\gamma) \cdot L_{we}^{\frac{7}{9}} \cdot D_{we}^{\frac{29}{27}} \quad \rightarrow \quad \textbf{max}$$

$$h_{min}(\gamma, D_{we}, L_{we}) = f_{h\_min}(\gamma) \cdot L_{we}^{0.02889} \cdot D_{we}^{0.99037} \quad \rightarrow \quad \textbf{max}$$

where:

$$f_{Cr}(\gamma) = 189.70078\,\gamma^{\frac{2}{9}} \cdot (1-\gamma)^{\frac{29}{27}} \cdot (1+\gamma)^{-\frac{1}{4}} \cdot \left[1 + 1.19303 \cdot \left(\frac{1-\gamma}{1+\gamma}\right)^{\frac{143}{24}}\right]^{-\frac{2}{9}} \cdot \left[\mathrm{floor}\left(\frac{2.70827}{\gamma}\right)\right]^{\frac{3}{4}}$$

and

$$f_{h\_min}(\gamma) = 3.264924\,10^{-3} \cdot \gamma^{-0.72889} \cdot \left[1 + 1.19303 \cdot \left(\frac{1-\gamma}{1+\gamma}\right)^{\frac{143}{24}}\right]^{0.02889} \cdot (1-\gamma)^{0.56037} \cdot (1+\gamma)^{0.4625} \cdot$$
$$\cdot \left[\mathrm{floor}\left(\frac{2.70827}{\gamma}\right)\right]^{0.0325} \cdot \min\left[\left(\frac{1-\gamma}{1+\gamma}\right)^{0.43}, 0.61557\right]$$

In order to perform the optimization, nine constraints were identified:

C1: Roller diameter D_{we} must be greater than or equal to a required value, D_{we_min}:

$$D_{we} \geq D_{we\_min} = 0.225 \cdot (D - d)$$

C2: Roller diameter D_{we} must be less than or equal to an imposed value, D_{we_max}:

$$D_{we} \leq D_{we\_max} = 0.275 \cdot (D - d)$$

C3: Difference between inner and outer ring thickness has to be greater than or equal to a minimum imposed value, δ_{io_min} = 2 mm:

$$\delta_i - \delta_o \geq \delta_{io\_min} = 2\,\mathrm{mm}$$

C4: Difference between inner and outer ring thickness has to be less than or equal to a maximum imposed value, δ_{io_max} = 5 mm:

$$\delta_i - \delta_o \leq \delta_{io\_max} = 5\,\mathrm{mm}$$

C5: Circumferential distance (tenon thickness) δ_c between rollers, measured on the pitch circle, has to be greater than or equal to a minimum value, δ_{c_min}:

$$\delta_c \geq \delta_{c\_min} = 0.16 \cdot D_{we}$$

C6: Circumferential distance (tenon thickness) δ_c between rollers, measured on the pitch circle, has to be less than or equal to a maximum value, δ_{c_max}:

$$\delta_c \leq \delta_{c\_max} = 0.25 \cdot D_{we}$$

C7: Roller length L_w has to be less than or equal to a maximum imposed value, L_{w_max}:

$$L_w \leq L_{w\_max} = 0.69 \cdot B$$

C8: The maximum hoop stress σ_{hoop} along inner ring cross-section must be less than or equal to the allowable hoop stress (for a maximum value of interference I = 0.3 mm) σ_{hoop_all} = 150 MPa:

$$\sigma_{hoop} \leq \sigma_{hoop\_all} = 150\,\mathrm{MPa}$$

where:

$$\sigma_{hoop} = 1.04 \cdot 10^5 \cdot \left[1 + \left(\frac{1}{1 + 2 \cdot \frac{\delta_i}{d}} \right)^2 \right] \cdot \frac{I}{d}$$

δ_i: inner ring minimum thickness (mm);
I: maximum interference (mm).

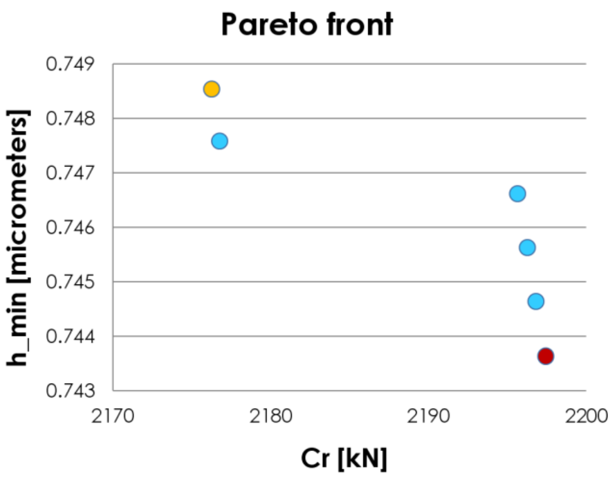
C9: The maximum Hertz stress σ_{H_i} along the most loaded roller-inner ring raceway contact has to be less than or equal to the allowable Hertz stress $\sigma_{H\_all} \approx 1300$ MPa

$$\sigma_{H\_i} \le \sigma_{H\_all} = 1300 \text{MPa}$$

where:

$$\sigma_{H\_i} = 603.144 \cdot \sqrt{\frac{p_C \cdot C_r}{L_{we} \cdot D_{we} \cdot (1-\gamma) \cdot \text{floor}\left(\frac{2.70827}{\gamma}\right)}}$$

In order to solve the above optimization program we have used a mono- and multi-objective optimization platform developed in-house and based on all sorts of Evolutionary Algorithms. The most representative individuals from the obtained Pareto front are presented in figure 5.



D_{pw} [mm]	D_{we} [mm]	Z	C_r [kN]	h_{min} [μm]
575	60	25	2176.269	0.74854
574	60	25	2176.816	0.74756
575	59	26	2195.744	0.74660
574	59	26	2196.307	0.74561
573	59	26	2196.869	0.74462
572	59	26	2197.432	0.74364

Fig. 5 - Representative individuals of the Pareto front

As one can observe from the plot in figure 5 the values of the minimum lubricant film thickness in the area of rolling contacts (roller-inner ring raceway in almost all cases) are very close together and definitely cover the asperities of the rough surfaces of the roller and raceways. So the chosen solution for the RKB NP 1092 next generation design will be that with the maximum basic dynamic radial load rating (red values in figure 5) and sketched in figure 6.

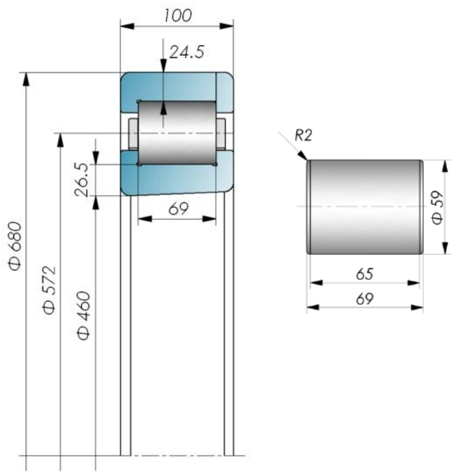


Fig. 6 - RKB NP 1092 next generation design

11. Conclusions

Over the past decades EAs have proved to be very powerful tools in the optimal design of all sorts of mechanical, electrical, hydraulic etc. complex systems. Although their use in the design of rolling-contact bearings is somewhat delayed and there are relatively few studies in this field, the results so far make us confident in their widespread use. The design of bearings with high load capacities, low weight, low friction losses, and high resistance to wear is a goal of any manufacturer of high-performance bearings. In addition, the use of EAs in the optimal design of rolling-contact bearings is justified because these algorithms are robust, flexible and able to deliver concrete and very good results in a reasonable period of time. For this reason, the RKB Bearing Industries Group has been using the Evolutionary Algorithms for many years, well aware that optimized products can make the difference in performance with the other producers. As regards optimal design, the current research trends at RKB focus most of all on the influence of the profile of the roller axial section and raceways cross-section on the basic dynamic load rating of the bearing.

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